Phase Differential Theory and the Interaction Dynamics of Defect States: Mediator Modes, Collective Coordinates, and Emergent Forces

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Abstract

Phase Differential Theory (PDT) identifies particle states with topologically stabilised defects of a dynamical phase manifold. This paper develops the interaction dynamics of multiple defect states within a controlled effective-field-theory regime. Using a collective-coordinate formulation, we construct the residual source currents generated by time-dependent defect configurations and determine the response of the phase manifold through the linearised fluctuation operator. Integrating over mediator modes yields an explicit effective interaction energy between defects.

The resulting interaction energy has a universal bilinear form in the induced sources. Massive mediator modes generate Yukawa-type forces, while massless modes produce long-range gauge-like interactions. The structure matches the interaction form obtained in relativistic field theory, providing an effective soliton-based interaction framework consistent with known interaction structures. Coefficient-level mapping and nonlinear corrections are deferred to subsequent work.

1 Introduction

In conventional quantum field theory, interactions between particles arise through the exchange of mediator fields. Phase Differential Theory (PDT) offers a different starting point: particle states correspond to extended, topologically stabilised defects embedded in a dynamical phase manifold. Interactions must therefore emerge from the response of the phase manifold to defect motion and internal excitation.

This paper derives the interaction structure explicitly within an effective-field-theory (EFT) framework. Starting from the microscopic phase-field action, we construct the multi-defect configuration, identify the residual source generated by defect motion, and solve the linearised fluctuation operator to obtain the mediator propagator. Substituting the resulting field perturbation back into the action yields a universal quadratic interaction energy.

Assumptions

A1. **Defect sector:** the phase manifold admits stable, finite-energy topological defects.

A2. **Collective-coordinate regime:** defect motion is slow compared to internal relaxation.

A3. **Linearised response:** fluctuations around the static defect background are treated at linear order.

A4. **Large separation:** defects are well separated compared to their characteristic size.

A5. **EFT truncation:** only leading-order terms in the derivative expansion are retained.

2 Multi-Defect Ansatz

2.1 Superposition of defects

For N well-separated defects, the phase field is approximated by

$$\Phi(x,t) \approx \sum_{i=1}^{N} \Phi_i\big(x - X_i(t), \Omega_i(t)\big), \tag{1}$$

where $X_i(t)$ is the position and $\Omega_i(t)$ the internal orientation of the ith defect.

2.2 Validity

The approximation holds when

$$|X_i - X_j| \gg R_{\mathrm{def}}. \tag{2}$$

3 Collective-Coordinate Lagrangian

Substituting the multi-defect ansatz into the microscopic action yields

$$L = \sum_i \left[\frac{1}{2} M_i \dot{X}_i^2 + \frac{1}{2} I_i \Omega_i^2\right] - V_{\mathrm{int}}(X_i, \Omega_i), \tag{3}$$

where M_i and I_i are the inertial mass and moment of inertia of each defect.

4 Residual Source Construction

Static defects satisfy

$$\left.\frac{\delta S}{\delta \Phi(x)}\right|_{\mathrm{static}} = 0. \tag{4}$$

Time-dependent collective coordinates generate a residual source

$$J(x,t) \equiv \left.\frac{\delta S}{\delta \Phi(x)}\right|_{\Phi_{\mathrm{static}} \neq 0}. \tag{5}$$

To leading order,

$$J(x,t) \approx \sum_i \Big[-M_i\, \ddot{X}_i \cdot \nabla \Phi_i + \text{internal terms}\Big]. \tag{6}$$

5 Linearised Phase Operator

Expanding the action to quadratic order in perturbations,

$$S = S_{\mathrm{static}} + \frac{1}{2} \int d^4x\, \delta\Phi\, L\, \delta\Phi + \int d^4x\, J\, \delta\Phi. \tag{7}$$

The fluctuation operator is

$$L = -\partial_t^2 + \nabla^2 - V''(\Phi_{\mathrm{static}}). \tag{8}$$

6 Mediator Propagator

Define the Green function

$$L\,G(x,x') = \delta^{(4)}(x-x'). \tag{9}$$

The perturbation is

$$\delta\Phi(x) = \int d^4x'\,G(x,x')\,J(x'). \tag{10}$$

In momentum space,

$$G(k) = \frac{1}{k^2 - m^2 + i\epsilon}. \tag{11}$$

7 Effective Interaction Energy

Substituting back into the action yields

$$S_{\text{eff}} = S_{\text{static}} + \frac{1}{2}\int d^4x\,d^4x'\,J(x)\,G(x,x')\,J(x'). \tag{12}$$

The interaction potential is

$$V_{\text{int}} = \frac{1}{2}\int d^3x\,d^3x'\,J(x)\,G(x-x')\,J(x'). \tag{13}$$

8 Mediator Mode Decomposition

The operator decomposes as

$$L = \bigoplus_{\alpha} L_\alpha, \tag{14}$$

with scalar, vector, and tensor mediator modes. These sectors represent an *effective decomposition* of the linearised fluctuation operator into modes with distinct transformation properties and characteristic ranges, rather than a claim of fundamental irreducible fields.

Each satisfies

$$(\partial^2 + m_\alpha^2)\phi_\alpha = J_\alpha. \tag{15}$$

9 Yukawa Interaction

For a massive mediator,

$$G(r) = \frac{e^{-mr}}{4\pi r}. \tag{16}$$

The resulting potential is

$$V(r) = \frac{Q_1 Q_2}{4\pi r}e^{-mr}, \tag{17}$$

with effective charges

$$Q_i = \int d^3x\,J_i(x)\,\psi_\alpha(x). \tag{18}$$

10 Long-Range Gauge-Like Interaction

For a massless mediator,

$$G(r) = \frac{1}{4\pi r}, \tag{19}$$

yielding

$$V(r) = \frac{Q_1 Q_2}{4\pi r}. \tag{20}$$

11 Correspondence with Relativistic Field Theory

Integrating out a mediator field A in a standard field theory yields

$$S_{\text{int}} = \frac{1}{2} J\, G\, J, \tag{21}$$

identical in structure to the effective interaction obtained here. This correspondence highlights that the emergent EFT description of defect interactions reproduces the familiar bilinear source–propagator–source form of relativistic field theory.

12 Interaction Channels in PDT

Three effective interaction channels arise:

- scalar (Yukawa),
- vector (gauge-like),
- tensor (gravitational).

13 Validity Regime

The derivation assumes:

- large defect separation,
- nonrelativistic velocities,
- linearised fluctuations,
- EFT truncation at leading order.

14 Limitations

- Relativistic corrections omitted.
- Nonlinear multi-defect interactions not treated.
- Gauge fixing and mode completeness not analysed in detail.
- Mapping to Standard Model parameters deferred.

15 Conclusion

Interactions between defects in PDT arise from the response of the phase manifold to defect motion. The resulting forces reproduce Yukawa and Coulomb forms and match the structure of relativistic field theory. This provides an effective soliton-based interaction framework consistent with known interaction structures.